

Jamming Transition in CA Models for Traffic Flow

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Abstract. The cellular automaton model for traffic flow exhibits a jamming transition from a free-flow phase to a congested phase. In the deterministic case this transition corresponds to a critical point with diverging correlation length. We present data from numerical simulations which suggest the absence of critical behavior in the presence of noise. The transition of the deterministic case is smeared out and one only observes the remnants of the critical point.

1 Introduction

The cellular automaton (CA) approach to traffic flow theory [1] has attracted much interest in recent years. Compared to the earlier attempts in modeling traffic flow (see e.g. [2] and references therein) CA models can be used very efficiently for computer simulations. This allows to perform realtime simulations of large networks [3].

One of the simplest models of that type is the model introduced by Nagel and Schreckenberg (NaSch model) [1]. This model is discrete in space and time. A discrete space means that the road is divided into L cells which can be occupied by a car or can be empty.

The model contains only a few parameters: The braking noise p , the maximum velocity v_{max} and, if one considers periodic boundary conditions, the average density ρ of cars. The update is divided into four steps which are applied in *parallel* to all cars. The first step (R1) is an acceleration step. The velocities v_j of each car $j = 1, \dots, N$ not already propagating with the maximum velocity v_{max} are increased by one. The second step (R2) is designed to avoid accidents. If a car has d_j empty cells in front of it and its velocity (after step (R1)) exceeds d_j the velocity is reduced to d_j . Up to now the dynamics is completely deterministic. Noise is introduced via the randomisation step (R3). Here the velocities of moving cars ($v_j \geq 1$) are decreased by one with probability p . The steps (R1)–(R3) give the new velocity v_j for each car j . In the last step of the update procedure the positions x_j of the cars are shifted by v_j cells (R4) to $x_j + v_j$.

For a given set of parameters v_{max} and p one can distinguish two different phases depending on the density [4,5,6,7]. For low densities each car can propagate with its desired velocity $v_{max} - p$ such that the flow is given by

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$J = \rho(v_{max} - p)$. For higher values of the average density jams occur and start and stop waves dominate the dynamics. Here we study the nature of the transition and the behaviour of the model at the transition point.

2 Order - Parameter

For a proper description of the transition one has to introduce an order-parameter (see also [8]) with a qualitative different behaviour in the two phases. Due to particle conservation one cannot use the density of cars, which would be the analogue to the magnetisation in the Ising model. Among several possible choices of an order-parameter the density of nearest neighbour pairs

$$m(\rho) = \frac{1}{L} \sum_{i=1}^L \langle n_i n_{i+1} \rangle \quad (1)$$

with $n_i = 1(0)$ if the site i is occupied (empty), is probably the simplest choice of a *local* order-parameter. One could think of alternative definitions, e.g. the density of cars with a distance less than v_{max} , but those results are in qualitative agreement with results obtained using (1). Fig. 1 shows the density dependence

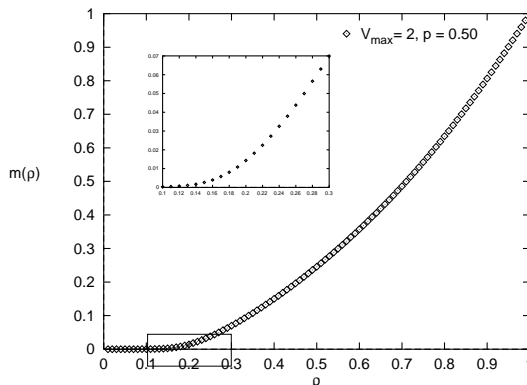


Fig. 1. Behavior of the order-parameter for a finite braking probability p . It does not vanish exactly for $\rho < \rho_c$, but converges smoothly to zero even for small values of the braking probability p .

of the order-parameter for $v_{max} = 2$ and $p = 0.50$. In the free flow regime the order-parameter vanishes and one observes a monotonous increase of m for higher values of the average density. At the transition regime m converges smoothly to zero if a finite value of the braking noise is considered. This behaviour is not the result of a finite system size, the data shown in Fig. 1 show no size dependence anymore. Due to the smooth convergence of the order-parameter it is difficult

to determine a transition density, but it is obvious that interactions between the cars occur already below the density of maximum flow.

In the deterministic limit ($p = 0$) the situation is different. Here one observes a sharp transition at $\rho_c = \frac{1}{v_{max}+1}$, the density of maximum flow.

3 Spatial Correlations

A striking feature of a second-order phase transition is the diverging length scale at criticality. In order to determine the length scale for a given density, we measured the density-density correlation function

$$G(r) = \frac{1}{L} \sum_{i=1}^L \langle n_i n_{i+r} \rangle - \rho^2. \quad (2)$$

Fig. 2 shows the behaviour of the correlation function near the transition density

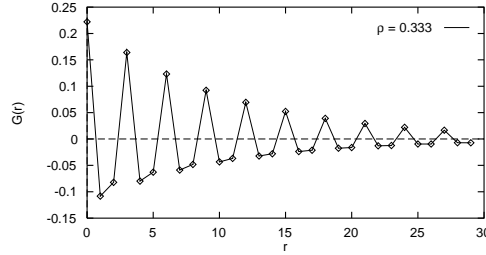


Fig. 2. Correlation function in the presence of noise ($v_{max} = 2, p = \frac{1}{128}$). The amplitude of the correlation function decays exponentially for all values of ρ .

for $v_{max} = 2$ and $p = \frac{1}{128}$. The correlation function is a damped oscillation with period $v_{max} + 1$. The amplitude is decaying exponentially for all values of the average density if a finite value of p is considered. In the deterministic limit one gets precisely at $\rho = \rho_c$ the following behaviour of the correlation function:

$$G(r) = \begin{cases} \rho_c - \rho_c^2 & \text{for } r \equiv 0 \pmod{v_{max} + 1} \\ -\rho_c^2 & \text{else} \end{cases} \quad (3)$$

Due to the exponential decay one can determine the density dependence of the correlation length $\xi(\rho)$ if $p > 0$.

Fig. 3 shows the behaviour of $\xi(\rho)$ for $v_{max} = 2$ and different values of p . The figure shows that the correlation length has a maximum ξ_{max} near the transition density, which is less pronounced for higher values of the braking noise.

In Fig. 4 the noise dependence of $\xi_{max}(p)$ is shown. The numerical data are in excellent agreement with

$$\xi_{max} \sim p^{-\frac{1}{2}} \quad p \rightarrow 0. \quad (4)$$

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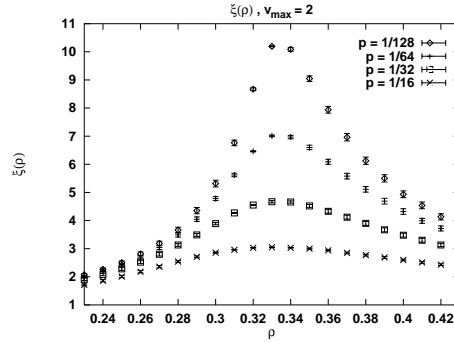


Fig. 3. Density dependence of the correlation length in the vicinity of the transition.

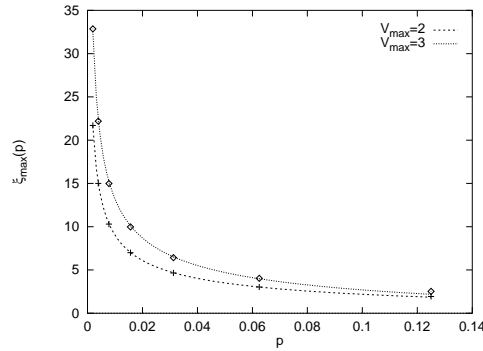


Fig. 4. Noise dependence of ξ_{max} for different maximum velocities. Independent of the maximum velocity, $\xi_{max}(p) \sim 1/\sqrt{p}$ holds in the limit $p \rightarrow 0$.

The divergence for $p = 0$ is consistent with the behaviour of $G(r)$ at the transition density, where one observes a constant amplitude of $G(r)$ and therefore an infinite correlation length. Finally it should be noted that the numerical results can be confirmed analytically for $v_{max} = 1$ [9].

4 Summary

We found evidence for the absence of criticality in the NaSch model in the presence of noise. For finite p the second order transition of the deterministic case is smeared out, similar to the situation of a equilibrium second order transition in a finite system. Here this effect is caused by the presence of noise, $p > 0$.

For small values of p one finds an ordering transition close to $\rho_c = 1/(v_{max} + 1)$. The ordering is due the parallel dynamics and can be found for all values of the maximum velocity, including $v_{max} = 1$. Larger values of the noise p favour the formation of jams and a tendency to phase separation occurs (see also [7,10,11])

if $v_{max} > 1$. The demixing of free flow and the jammed regime becomes more and more exact if larger values of v_{max} and p are considered, because interactions between cars become rare in the free flow regime of the system.

The effect of fluctuations is twofold: First, one already finds blocked cars for densities below ρ_c and second, exact phase separated states are not stable, e.g. one observes the spontaneous formation of jams in the outflow region of a megajam.

Finally we discuss the deterministic limit $p = 1$. Here we find metastable states for densities below $\rho_c = 1/3$ if $v_{max} > 1$. These metastable states consist of moving cars with at least two empty cells in front. They are metastable in the following sense: If a car stops due to a perturbation of the system, all other cars have to stop after $O(L)$ lattice updates. These metastable states can only be found for $v_{max} > 1$, because for $v_{max} = 1$ obviously the cars cannot move. The lack of metastable states for $0 < p < 1$ is consistent with our observation that *exact* phase separation is absent for this cases.

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